SIX-PORT MEASUREMENT TECHNIQUE: PRINCIPLES, IMPACT, APPLICATIONS

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1. Summary
The paper reviews the development of the six-port measurement technique since the time it was introduced in 1977 until present days. Working principles of six-port reflectometer and six-port network analyzer are explained. The technique is put in context with other scattering parameter measurement principles; the advantages and drawbacks are discussed. Some typical SPR implementations are presented. Basic six-port calibration methods are outlined; general significance of TRL method is accented. Accomplishments achieved in the Department of Radio and Electronics of FEEIT STU Bratislava are presented. Current standing of the technique is assessed.

2. Introduction
The official birthyear of the six-port measurement technique is 1977, when three fundamental papers [1] – [3] and several accompanying papers were published in the December issue of the IEEE Transactions on Microwave Theory and Techniques. Although the inventors, Glenn F. Engen and Cletus A. Hoer of the National Bureau of Standards (now National Institute of Standards and Technology), Boulder, Colorado, USA, had published papers with partial ideas and used the term previously, these articles presented for the first time a complete and unified theoretical background and offered guidelines for optimum six-port design.

The six-port technique is a method of network analysis, i.e. that of scattering parameters measurement: either only of reflection coefficient, in which case we speak about six-port reflectometer (SPR) or both reflection and transmission coefficients, in which case we speak about six-port network analyzer (SPNA). Although the principle of SPNA will be explained, this paper concentrates on six-port reflectometers.

3. Scattering Parameters
While in low-frequency applications the signals we are concerned with are voltages and currents, in RF and almost exclusively in microwave applications the signals are described by wave variables, linked with physical waves traveling along transmission lines (Fig. 1). For waves traveling toward a device under consideration (incident waves) these variables are denoted \(a\); for waves traveling outward (transmitted and reflected waves) these variables are denoted \(b\).

\[
\begin{array}{ccc}
\text{Circuit} & \text{Ref. plane 1} & \text{Ref. plane 2} \\
\text{a}_1 & \text{b}_1 & \text{b}_2 \\
\text{b}_1 & \text{a}_2 & \text{a}_1 \\
\end{array}
\]

Fig. 1: A microwave circuit (2-port)

Magnitude of a wave variable is derived from mean power \(P\) carried by the corresponding physical wave:
This is because $P$ is an unambiguous quantity while, apart from TEM waves (propagating e.g. in coaxial lines), voltage and current cannot be uniquely defined in microwave circuits.

**Phase.** A wave variable must be related to a certain plane perpendicular to the direction of propagation (reference plane). The phase of a wave variable is then equal to that of the corresponding physical wave at that plane (e.g. of the electric field strength).

In cases where voltage can be unambiguously defined (e.g. in lumped-element circuits or in transmission lines with TEM waves) there is the following relation between the voltage and wave variable:

$$a = \frac{V}{\sqrt{2Z_0}}$$

$V$ is the voltage phasor (complex amplitude) of an individual wave at a given plane and $Z_0$ is chosen real reference impedance (e.g. the characteristic impedance of the transmission line).

![Fig. 2: Distribution (scattering) of incident wave $a_1$ in a microwave circuit](image)

The network under consideration (device under test - DUT) may have one or more (generally $n \geq 1$) input/output ports (Fig. 2). A reference plane is assigned to each port at which the wave variables are defined. So, there are $n$ input signals (incident waves) $a_i$, $i = 1...n$, and $n$ responses $b_j$, $j = 1...n$. Any linear n-port is therefore characterized by $n^2$ transfer functions, relating the responses to the stimuli. The transfer functions are denoted $S_{ij}(f)$ and called scattering parameters or S-parameters of the n-port. The complete relation is given by the matrix equation

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$
The S-parameters can be classified as

- reflection coefficients \( \Gamma_i = S_{ii} \), when the response is a wave traveling out of DUT in the same port as the stimulus (reflected wave), or
- transmission coefficients \( t_{ji} = S_{ji} \), \( j \neq i \), when the response is a wave traveling out of DUT in another port.

4. Network Analyzers

The network analyzer (NA) is an instrument to measure scattering parameters. Two distinct measurement principles can be differentiated:

- Wave separation method (conventional NA)
- Interference method (slotted line, six-ports)

**Wave Separation Method**

This is conceptually straightforward, definition-based measurement. Practically all commercially available NA work on this principle. A single stimulus (incident wave) is injected into one of the ports and the responses at all ports are observed and compared with the stimulus.

For the measurement of reflection coefficient, the incident and reflected waves must be somehow separated since they appear at the same port. Directional couplers or directional bridges (Wheatstone bridges) are used for this purpose.

To arrive at a scattering parameter, we must compute

- the ratio of amplitudes
- the phase difference

of the corresponding response and the stimulus. Obtaining the phase difference is a major complication. Heterodyne dual-conversion receivers phase-locked to the stimulus are employed for this purpose.

The principal block diagram of a conventional NA is in Fig. 3 where DUT (device under test) is the object being measured.

![Fig. 3: Conventional, wave separation-based network analyzer](image)

**Interference Method**

The interference method does not separate the stimulus from the response. On the contrary, several controlled linear combinations of the two waves are created in the measurement system, and the resulting amplitudes are observed. Using these amplitude-only data, both modulus and phase of scattering...
parameters can be obtained. Six-port method belongs to this category. The required number of wave combinations is optimally three plus a sample where the stimulus prevails (reference signal). The wave combinations can be

- made and observed simultaneously at various points (ports) of the measurement system, or
- created and observed sequentially at the same port (multistate or switched systems). The slotted line is an example of such a system; here the incident and reflected wave are combined along the line, giving rise to standing wave pattern (Fig. 4).

Care must be taken with the multistate systems because certain conditions must be satisfied for their applicability. Loosely speaking, the external circuits must not notice there is something happening inside the system when the state is changing (switching, moving a probe).

![Diagram](https://via.placeholder.com/150)

**Fig. 4: Slotted line**

The question now may be raised: Is then the slotted line a SPR? The answer is: What makes a device SPR is not only its hardware implementation but also the way the measured data are processed. If the mathematics of SPR is applied to voltages obtained from the detector of a slotted line probe (taken e.g. at the positions designated by the red circles), then the answer may be yes.
In SPNA, the interference principle is even more noticeable: the stimuli are applied at both ports of DUT simultaneously.

5. Six-Port Reflectometer

![Six-port reflectometer diagram](image)

Fig. 5: Principal block diagram of six-port reflectometer

The basic building block of the SPR is a linear six-port (Fig. 5), one port of which is terminated in a device under test (DUT), one port is connected to the signal source (GEN), and the remaining four ports are connected to power detectors (D1 – D4). DUT is characterized by the reflection coefficient $\Gamma$. The state of the six-port is determined by 12 complex wave variables $a_i, b_i, i = 1...6$. The variables are not independent; they are mutually coupled by scattering parameters of the six-port, i.e. by six equations incorporated in (1). Moreover, since the detector ports are terminated in defined (although not inevitably known) loads – detectors, additional four restraints are added of the form

\[(2) \quad a_i = \Gamma_{Di} b_i \quad i = 1...4\]

where $\Gamma_{Di}$ is reflection coefficient of $i$-th detector. Hence, there are ten constraints for the twelve variables $a_i, b_i$. The system has therefore only two degrees of freedom. This means that only two of the waves can be chosen arbitrarily; the rest can be expressed as linear combinations (superpositions) of the two. For sake of SPR theory, it is useful to choose the wave $b \equiv b_5$ incident on DUT and the wave $a \equiv a_5 = \Gamma b$ reflected from DUT as the independent variables. The waves incident on the detectors can then be expressed as

\[(3) \quad b_i = A_i a + B_i b = b A_i \left(\frac{a}{b} \frac{B_i}{A_i}\right) = b A_i (\Gamma - q_i) \quad i = 1...4\]

where $A_i, B_i$ are complex quantities characterizing SPR (they depend only on S-parameters of the six-port and reflection coefficients of the detectors),
\[ \Gamma = a/b \]
is the sought reflection coefficient of the DUT, and the complex numbers
\[ q_i = -B_i/A_i \]
are termed \textit{q-points} of the SPR. Detector outputs are supposed to be proportional to input RF powers:
\[ P_i = |b_i|^2 = |A_i a + B_i b|^2 = P_0 |A_i|^2 |\Gamma - q_i|^2 \]

Here \( P_0 = |b|^2 \) is the power incident on DUT (in many cases also of metrologist’s interest). One of the detectors, called \textit{reference detector} (here denoted as D4) should ideally, although not necessarily, respond only to wave \( b \), incident on the DUT: this would lead to \( q_4 \) approaching infinity. It is therefore more appropriate for the reference detector to write (5) in an alternative form
\[ P_4 = P_0^2 |B_4|^2 |d\Gamma + 1|^2 \]
where \( d = A_4/B_4 = -1/q_4 \). For an ideal reference port, \( d = 0 \).

Introducing normalized powers \( p_i = P_i/P_4 \), one can write
\[ p_i = \frac{P_i}{P_4} = C_i \left( \frac{\Gamma - q_i}{d\Gamma + 1} \right)^2 \]

where \( C_i = |A_i/B_i|^2 \). The three equations (7) are often called the \textit{working equations} of the six-port reflectometer. They relate reflection coefficient \( \Gamma \) of DUT to measured quantities – detector powers. As seen, SPR is fully characterized by 11 parameters: four complex quantities \( (q_i, d) \) and three real scaling factors \( C_i \) (note that the parameters are frequency-dependent). These or other derived set of 11 parameters are called \textit{calibration constants}. They can be obtained in the process of six-port \textit{calibration}, in which a set of known loads (calibration standards) are connected in place of DUT and the detector responses are recorded. To arrive at the calibration constants from such data may be quite a formidable task. Once the calibration constants are known, the measurement consists in

- connecting DUT
- measuring detector powers
- solving three simultaneous nonlinear equations (7) for the unknown \( \Gamma \).

Graphically, each of the equations represents a circle in the \( \Gamma \)-plane (often called a \textit{q-circle}). The situation is best illustrated for the case of an ideal reference port \( (d = 0) \), when the equations (7) simplify to
\[ p_i = C_i |\Gamma - q_i|^2 \quad i = 1...3 \]

and represent circles with centers \( q_i \) and radii \( r_i = \sqrt{p_i/C_i} \), the radii being proportional to the square root of detector powers. Each circle represents a set of possible values of \( \Gamma \) satisfying the particular equation. The sought solution must be the common intersection of the three circles (Fig. 6). The shaded area (unit circle) corresponds to all passive loads.
It is evident that the q-points are essential SPR parameters since reflection coefficient can be accurately obtained only when q-points are properly positioned.

Note that the system is **overdetermined**, i.e. one of the three circles only decides between two possible intersections obtained from the other two circles. This redundancy inherent in SPR technique adds to measurement accuracy and helps detect any inconsistencies.

In the general case \( d \neq 0 \) the three-circle interpretation still holds, only the formulas for the centers and radii are more complex.

Analytically, the solution \( \Gamma \) is usually sought as the intersection of the three common cords \( a_{12}, a_{23}, \) and \( a_{31} \) (any two of them are sufficient). This leads to the simple formula

\[
\Gamma = \frac{c_1P_1 + c_2P_2 + c_3P_3 + c_4P_4}{m_1P_1 + m_2P_2 + m_3P_3 + P_4} + j \frac{s_1P_1 + s_2P_2 + s_3P_3 + s_4P_4}{m_1P_1 + m_2P_2 + m_3P_3 + P_4}
\]

or, in terms of normalized powers,

\[
\Gamma = \frac{c_1p_1 + c_2p_2 + c_3p_3 + c_4}{m_1p_1 + m_2p_2 + m_3p_3 + 1} + j \frac{s_1p_1 + s_2p_2 + s_3p_3 + s_4}{m_1p_1 + m_2p_2 + m_3p_3 + 1}
\]
where the 11 real parameters \( c_1 \ldots c_4, s_1 \ldots s_4 \), and \( m_1 \ldots m_3 \) are another form of calibration constants; they can be expressed in terms of \( q_i, d, C_i \). Equation (9) gives a reasonable solution (Fig. 7) also for the practical case of the circles not intersecting in a single point (even for the circles not intersecting at all).

**Power Measurement**

A special potential of SPR is its capability also to measure incident power \( P_0 \). This is enabled by Eq. (6), which can be converted to

\[
(10) \quad P_0 = K (m_1 p_1 + m_2 p_2 + m_3 p_3 + P_x) = KP_x (m_1 p_1 + m_2 p_2 + m_3 p_3 + 1)
\]

where \( m_i \) are the same as in (9). An additional constant \( K \) must be obtained by an extra calibration step consisting in incident power measurement with an arbitrary load (e.g. a thermistor mount) in place of DUT.

Knowing \( P_0 \) and \( \Gamma \), also the reflected power \( P_r \) and the power \( P_a \) absorbed in load (an antenna or a chicken in a microwave oven) can be computed:

\[
P_r = P_0 |\Gamma|^2 \quad P_a = P_0 - P_r = P_0 \left(1 - |\Gamma|^2\right)
\]

**Optimum SPR**

An optimum SPR is one that is the least sensitive to detector power measurement error. The outlined graphical interpretation of working equations indicates that this is closely linked with q-point positions. Intuitively:

- The best case appears when the q-points are arranged symmetrically around the reflection coefficient to be measured.
- A fatal situation occurs when two q-points coincide or when all are centered on a single line; this results in two indistinguishable solutions (Fig. 8).
- An ill-conditioned, although not fatal, case occurs when \( \Gamma \) is very close to a q-point. This is because in such case the detector power \( P_i \) approaches zero and the q-circle radius is in fact determined merely by noise fluctuations. The same noise level less affects greater radii.
- An unfavorable case also occurs for reflection coefficients on a line connecting two circle centers where q-circles do not intersect but only touch. This results in excessive sensitivity to circle radii variations (Fig. 9).

The criteria for an optimum SPR can then be summed up as follows:
1. Reference detector should respond to the wave incident on DUT.
2. Phases of q-points should differ by 120°.
3. Magnitudes of q-points should be approximately 2.

The latter two criteria can also be formulated such that the q-points are vertices of an equilateral triangle touching or encompassing the unit circle (Fig. 10).

SPR design then consists in determining the q-points of a given structure and trying to approximate the above criteria in as wide a frequency range as possible or required.

Fig. 8: Fatal situation: all Q-circle centers in-line

Fig. 9: Touching Q-circles: high sensitivity to power measurement error
6. Dual Six-Port Network Analyzer

Two six-port reflectometers can be used in the configuration shown in Fig. 11 to measure all S-parameters of any linear two-port [3]. Such arrangement is usually called dual six-port network analyzer. A six-port reflectometer (SPR1, SPR2) is connected to each port of the DUT. Both reflectometers are simultaneously fed from the same signal source using a power divider PD. The complex ratio $a_2/a_1$ of the input waves can be varied by means of the attenuator AT cascaded with the phase shifter PS. The waves $a_i, b_i$ are interrelated by the scattering equations

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

where $S_{ij}$ are the sought scattering parameters of DUT.
Fig. 11: Dual six-port network analyzer

SPR1 and SPR2 measure the complex ratios

$$\Gamma_1 = \frac{b_1}{a_1} = S_{11} + S_{12} \frac{a_2}{a_1}$$

$$\Gamma_2 = \frac{b_2}{a_2} = S_{22} + S_{21} \frac{a_1}{a_2}$$

respectively. Eliminating the ratio $a_2/a_1$ leads to the equation

$$\Gamma_2 S_{11} + \Gamma_1 S_{22} - D = \Gamma_1 \Gamma_2$$

for three unknowns: $S_{11}$, $S_{22}$, and the determinant $D = S_{11}S_{22} - S_{12}S_{21}$. The unknowns can be obtained by solving a simultaneous set of at least three equations (11). The equations are generated by various settings of the phase shifter PS and attenuator AT. The settings need be neither known nor reproducible: they do not enter into the equations. If the DUT is known to be reciprocal, $S_{12} = S_{21}$, can be expressed from $D$, hence the task is completed. If the DUT is not known to be reciprocal, some more work is to be done: three different reciprocal two-ports with approximately known $a_2/a_1$ ratio must be measured to calibrate the same three settings of the AT+PS combination. The two-ports can be two transmission line sections with approximately known lengths and a “thru”-device: direct connection of SPR1 and SPR2. Imperfect repeatability of the AT+PS settings affects only phases of $S_{12}$ and $S_{21}$.

7. Calibration

Calibration is the process of obtaining the 11 calibration constants of a SPR in whatever form they may be expressed. A considerable effort has been devoted to the development of calibration procedures. Some methods (notably TRL and its LRL modification) are applicable also to conventional network analyzers.
In the process of calibration, a set of terminations are connected in place of DUT and the corresponding detector powers are recorded. The terminations may be known (calibration standards) or unknown (auxiliary loads). The detector powers and reflection coefficients of the terminations are substituted to the SPR working equations (7). The obtained set of equations is solved for the unknown calibration constants (reflection coefficients of the unknown loads are obtained as a by-product).

The calibration loads can be classified as follows:

1. Fully known loads (calibration or impedance standards). Their reflection coefficients enter SPR equations as known quantities. Their knowledge is therefore critical since it directly affects the accuracy with which the calibration constants are obtained hence the resulting SPR measurement accuracy. Generally, there is an effort to develop calibration methods which use use as few standards as possible.

2. Unknown loads. Their parameters enter SPR equations as unknown quantities hence do not affect SPR measurement accuracy. Each such load adds two new unknowns (the real and imaginary parts of its reflection coefficient) but generates three equations (7), hence the net gain is one. The unknown loads therefore reduce number of required standards. The loads can be unknown yet not arbitrary: they must be chosen such that the generated equations are independent (e.g. the same equations are generated by a short circuit and a shorted transmission line half-wavelength long).

3. Partially known loads. They serve in principle as unknown loads but the approximate knowledge of some of their parameters (e.g. reflection coefficient, length) helps decide between more mathematically possible solutions. They may also serve for finding an approximate solution used as first guess in numerical procedures.

4. Sliding loads. The property is exploited that when the load is slid the reflection coefficient moves along a circle in the \( \Gamma \)-plane. It usually leads to complicated mathematical procedures.

Typical calibration loads are

- Matched (non-reflecting) termination (fixed or sliding); often used as a standard
- Sections of transmission lines (shorted or open); often used as standards
- Attenuators terminated by the above sections of transmission lines
- Capacitors or inductors (for lower frequencies)

An example of waveguide calibration set (8.2 – 12.4 GHz) consisting of a sliding matched load and four offset shorts is shown in Fig. 12.
The mathematics of calibration is simpler when more known standards are employed. However, the accuracy suffers since it depends on uncertainties of the many standards. First calibration methods, e.g. [4], used as many as 7 standards. An extra inconvenience of using many loads is a limited frequency range.

Other six-port calibration methods have been developed that use only four standards, e.g. offset shorts [5], [6]. Benefit of using a fixed or sliding matched termination as 5th standard may be improved accuracy of small reflection coefficient measurements [7]. We have also contributed to these methods ([8]–[10]); we use them to obtain initial guess for the procedure outlined next.

**Six-Port To Four-Port Reduction**

Perhaps the most ingenious method has been developed in 1978 by Engen [11]. It is mathematically a two-step procedure. The first step is known as six-port to four-port reduction, the second step is calibration of the equivalent four-port.

The first step uses the fact that the system is overdetermined: i.e. one of the three normalized powers only decides between two possible solutions obtained from the other two normalized powers. Consequently, the normalized powers are not independent: they are subject to a constraining relation

\[
F(p_1, p_2, p_3, a, b, c, \zeta, \rho) = 0
\]

containing five out of the 11 SPR calibration constants (here denoted \(a, b, c, \zeta, \rho\)). The constants can be obtained by solving a set of at least 5 simultaneous equations (12), which can be generated by measuring \(p_i\) for at least 5 arbitrary unknown loads\(^1\). The equations are nonlinear, hence a numerical iteration must be employed and a good initial solution must be available. The initial solution can be obtained by one of the above-mentioned methods using the same loads, treating them in this case as known standards.

Having these constants, a complex ratio \(w = \frac{b_1}{b_4}\) can be computed for DUT connected to SPR:

\[
w = G(p_1, p_2, p_3, a, b, c, \zeta, \rho)
\]

This ratio is linked with the true reflection coefficient \(\Gamma\) of DUT via the formula

\[
w = E_d + \frac{E_i \Gamma}{1 - E_m \Gamma}
\]

The complex quantities \(E_d, E_i, E_m\), represent the remaining 6 calibration constants. When known, the sought \(\Gamma\) can be computed from the inverse of (14).

Eq. (14) is formally identical with the relation between the actual and measured reflection coefficients in conventional 4-port vector reflectometers, when the measurement is biased by the systematic directivity error \(E_d\), tracking error \(E_i\), and test port mismatch error \(E_m\). (This is the reason for the first calibration step to be called six-port to four-port reduction.) Consequently, as the second calibration step, all methods developed for the calibration of conventional reflectometers (i.e. for determining of \(E_d, E_i, E_m\)) can be applied. The simplest is the method which uses three standards, e.g. a short (\(\Gamma_1 = -1\)), an open (\(\Gamma_2 = +1\)), and a matched termination (\(\Gamma_3 = 0\)). Connecting these standards, we measure the values \(w_1, w_2\) and \(w_3\), respectively. Inserting \(\Gamma_i\) and \(w_i\) into (14) yields three equations, from which the unknowns \(E_d, E_i, E_m\) can easily be solved. (Actually, these standards need not be connected since they are normally among those used in the previous calibration step: only the data recorded for them are reused.)

Now, the calibration is complete.

**Thru-Reflect-Line Calibration**

The Thru-Reflect-Line (TRL) method [12] serves for complete calibration of dual six-port network analyzers. It requires connection of only three calibration devices, among them only one standard. The devices are (Fig. 13)

\(^1\) Methods like this which do not need standards (or need only a reduced number of standards) are called self-calibration methods.
• **THRU**: Direct connection of the two test ports.
• **REFLECT**: An unknown load with high reflection coefficient, connected successively to SPR1, then to SPR2.
• **LINE**: A piece of non-reflecting transmission line, optimally a quarter-wavelength long ($90^\circ$).

![Fig. 13: TRL calibration connections](image)

The only standard required is a length of precision transmission line or waveguide, which is the most accurate impedance standard available. The only condition imposed on the line is that it be *non-reflecting*; only this affects the analyzer’s measurement accuracy. This is why TRL is perhaps the most accurate calibration method invented.

Mathematically, the calibration consists of two steps:

1. As the first step, a six-port to four-port reduction is performed for the two six-port reflectometers. This requires no extra loads since the required equations can be generated by various settings of attenuator – phase shifter combination for the three calibration devices.
2. As the second step, both equivalent four-ports are calibrated. The mathematics of the procedure is too extensive to be explained here; the reader is referred to [12].

The second step of TRL calibration can also be applied to conventional network analyzers. In fact, all high-precision network analyzers use it. In this way TRL profoundly influenced the network analysis in general and represents perhaps the most significant contribution the six-port technique offered.

LRL (Line-Reflect-Line) calibration method differs from TRL only in that another line section is used instead of the THRU connection [13]. This makes the method more practical, especially when the test port connectors of SPR1 and SPR2 are of the same sex (THRU connection is impossible) or when the LINE length is too short to be realizable. A *difference* between the two line lengths should be optimally a quarter-wavelength.
8. Six-Port Reflectometer Implementations

This section is a gallery of various SPR implementations as well as an overview of the work done by our research group. There exist many more structures than those presented below: only lack of space forbids their mentioning.

It is worth bearing in mind in designing various implementations that a SPR structure must provide at least one phase shift different from $0^\circ$ or $180^\circ$, otherwise all q-points would lie on a single line.

**SPR with Directional Couplers**

These are the “classical” types, suggested by the SPR inventors [2]. They use 4-port directional couplers (Fig. 14) and power dividers. An important property of a directional coupler is that, as a response to an input wave, the wave emerging from the coupled arm is phase-shifted by $-90^\circ$ with respect to the wave emerging from the direct arm. This provides the required phase shift different from $0^\circ$ or $180^\circ$.

\[ b_3 = A a_1 - jB a_2 \]
\[ b_4 = A a_2 - jB a_1 \]

**Fig. 14: Wave distribution in a directional coupler**

**Fig. 15: Schematic diagram of the X-band waveguide SPR with directional couplers**

Fig. 15 and Fig. 16 show the first SPR realized in our laboratory. It was developed for X-band (8.2-12.4 GHz) in 1985. The medium was R-100 rectangular waveguide (22.86 mm x 10.16 mm). The SPR used precision high-directivity multihole couplers and zero-bias Schottky diode detectors. The theoretical q-point distribution, also shown in Fig. 15, reasonably well approximates the optimum. The SPR proved to be quite accurate: the reflection coefficient measurement uncertainty was found about 0.01. Its main
disadvantages were bulkiness (1.2 m x 0.3 m) and, which was then not recognized, excessive temperature dependence.

**Microstrip SPR with Directional Couplers**

To decrease dimensions, a microstrip clone of the waveguide SPR has been developed and tested in 1986 [15], consisting of only 3-dB Lange couplers and Wilkinson power dividers (Fig. 17).

![Fig. 17: X-band microstrip SPR layout](image)

The SPR was built on a 50 mm x 50 mm alumina substrate. Higher reflections from the microstrip couplers and power dividers compared to their waveguide counterparts completely corrupted SPR parameters and made the whole effort fail. The system was nicknamed Rebel alias Reactance Madhouse.
Three-Probe Six-Port Reflectometers

The three-probe six-port reflectometers turned out to be very successful and suitable for many practical applications requiring a medium bandwidth (up to 1 octave). The first type, completed in 1989, was an X-band waveguide reflectometer nicknamed Wasp [16]. Its principal diagram is shown in Fig. 18. The SPR consists of a waveguide section with a directional coupler (DC) feeding the reference detector D4 and three probes, spaced electrically by 60° at the center frequency of the required band. The probes respond equally to the waves travelling in both directions, which results in unit q-point magnitudes. The phase angle of a q-point is proportional to the distance the wave must travel from the probe to DUT and back to the probe. Given the 60° probe spacing, the q-point phases differ by 120°, which is the optimum case. With varying frequency, the q-points rotate, each with different speed, until two of them are too close to each other. This limits the operation bandwidth. Practically, the angular spacing should not drop below about 40°.

Fig. 19 shows the photograph of the realized SPR.
Using two reflectometers of this type, an X-band dual six-port network analyzer was completed and successfully tested [17] but the work discontinued in this direction.

SPR of similar type [18], named Homer, has been developed for high-power industrial applications at 2.45 GHz, handling powers of up to 30 kW (Fig. 20). The system is an autonomous unit constructed on a broad wall of R26 waveguide (86.36 mm x 43.18 mm). It contains its own single-board computer and communicates with external devices via RS232 or CAN bus. Very important for this application is the power measurement capability of SPR. Because of a free-running magnetron as signal source, the device must incorporate a frequency counter.
A miniature low-power three-probe SPR implemented as a combination of microstrip and lumped-element technology has been developed for the frequency range 2.2 – 2.7 GHz [19]. The SPR serves for the purpose of industrial applicators design. The circuit is built on a 35 mm x 35 mm Rogers TMM-6 substrate (εᵣ = 6), using 0805 SMD resistors and capacitors (Fig. 21). The main microstrip line is tapped by means of resistors at three points separated by approximately 60° at 2.45 GHz. Resistive coupling has been chosen for dimensional reasons. In addition, overlapping the edge-coupled directional coupler with the 3-probe section enabled to reduce the dimensions of the structure by the factor of nearly two. The length of the coupled section (60°) was chosen arbitrarily to conform the physical layout requirements. The directivity of such shortened coupler could be maximized by varying the impedance terminating the unused arm (a series RL combination).

![Microstrip three-probe SPR for 2.2 – 2.7 GHz](image)

A similar principle has been used in the high-power SPR [20] covering the 900 MHz ISM band, handling CW powers up to 100 kW (Fig. 22, Fig. 23). The transmission medium is the bulky R9 waveguide (247.66 mm x 123.86 mm). A two-probe directional coupler of our own proposal [21] has been used to couple signals from the waveguide to a 3-probe microstrip SPR realized on Rogers TMM-10 substrate (εᵣ = 9.3). This enabled to substantially reduce the dimensions of SPR (microstrip probe spacing is about 20 mm while waveguide probe spacing would be 75 mm).
Switched-Reflector Six-Port Reflectometers

The concept of switched-reflector SPR, where the number of detector ports is reduced to one, was originally conceived at Warsaw Polytechnics [22]. The principle is that, using switchable phase and amplitude modulators within the SPR structure, all combinations of the incident and reflected wave necessary for optimally distributed q-points can be sequentially created at a single detector port. Inspired by this, we have suggested and both theoretically and experimentally investigated several microstrip structures, containing two detectors [23]. An example is shown in Fig. 24. The reflectometer consists of:

- Dual directional coupler (DC1, DC2)
- Two detectors (D1, D2)
- Reflection phase modulator (RPM), consisting of three switchable circuits with reflection coefficients $\Gamma_i$ (most simply open-ended microstrip sections with lengths differing by 60°)

![Diagram of two-detector switched SPR](image)

**Fig. 24: Two-detector switched SPR**

D2 is the reference detector sampling the incident wave $b$. D1 responds to the combination of wave $a$ reflected from DUT and the wave reflected from RPM. This wave is proportional to $b$, only its phase depends on the state of RPM. By switching RPM to its three states, the three required combinations of $a$ and $b$ are obtained. Magnitudes of q-points depend on the coupling of DC1 and DC2; phase differences of q-points are equal to phase differences of $\Gamma_i$. One-decade bandwidth can be obtained using broadband RPM devised by Morawski and Zborowska [24].

A different type of switched SPR has been suggested at Czech Technical University [25]. It makes convenient use of existing scalar network analyzers but connects a multistate circuit, called perturbation two-port (PTP), between DUT and analyzer’s reflectometer head.

![Diagram of creating combinations using a perturbation 2-port](image)

**Fig. 25: Creating combinations of incident and reflected waves using a perturbation 2-port**

Switching between the internal states of the PTP changes its S-parameters, which, if properly designed, creates the required combinations of the incident and reflected wave at the reflectometer’s detector (Fig. 25). Except one of the states, the perturbation circuit must be partially reflective; this supplies a sample of
the incident wave to the detector. However, the same requirement also causes that, strictly speaking, the six-port theory does not directly apply.

Another type of the switched SPR will be presented in the section dealing with lumped six-ports.

An advantage of switched six-port reflectometers is their simple construction. The main disadvantages are reduced speed due to sequential measurement of detector voltages (impossibility to sample pulsed signals) and tendency to an additional error caused by the fact that SPR theory is not rigorously applicable.

**Dual-Generator Six-Port Reflectometers**

The phase shift different than 0° and 180° that is required for SPR operation can also be realized in a purely resistive structure if fed from two mutually synchronized, phase-shifted signal sources (Fig. 26). The case has been theoretically analyzed and experimentally verified with direct digital synthesis (DDS) generator at frequencies (0 – 10 MHz) [26]. A feasibility in the range 2 – 27 GHz using a directional coupler as phasing device has been confirmed by simulation. The idea has then been locked to drawer.

![Fig. 26: A dual-generator SPR](image)

9. **Lumped Six-Port Reflectometer**

This section is devoted to a special, extremely wideband SPR type which may contain only lumped elements and, consequently, lends itself to even monolithic integration. While typical bandwidths achievable by various SPR types range from less than one octave to about 10:1, this type can achieve bandwidths of up to 1000:1. The structure was invented in 1986. Its core is the Wheatstone bridge [27], [28]. There is a normal and multistate modification of the SPR. At least one institution (ENST Paris) still develops the ideas.

Basic circuit diagram of the SPR in its multistate modification is shown in Fig. 27. The device can conveniently be split into a power divider and a combination circuit.

**Power divider** provides the reference detector D2 with a sample of incident wave \( b \). Reflected wave \( a \) does not penetrate to the detector because the source of \( a \) (DUT) and D2 are in mutually isolated diagonals of a balanced bridge (\( R_a R_b = Z_0^2 \)).

Provided the bridge is balanced and D2 matched, the reflection coefficient seen at the divider input is proportional to that of DUT:

\[
\Gamma_D = t^2 \Gamma
\]

where

\[
t = \frac{R_b}{R_b + Z_0}
\]

is transmission coefficient between the divider input and the test port.
Combination circuit provides three linear combinations of the wave incident on and reflected from DUT; these determine the q-points. The two upper arms (1, 2) of the combination circuit are equal. The lower left-hand arm (3) serves as reflector: a circuit with high reflection coefficient \( \Gamma_R \), switchable between three values \( \Gamma_i \) (\( i = 1...3 \)) with differing phase angles. It can be proved (and it is the basic idea behind this SPR type) that the wave incident on detector D1 is a superposition with the same weight but opposite polarity of the waves reflected from the two lower arms (3, 4):

\[
\begin{bmatrix}
\Gamma - \Gamma = \Gamma - \Gamma = \Gamma - \Gamma
\end{bmatrix}
\]

This is actually a six-port equation yielding the q-point

\[
q = \Gamma_R / t^2
\]

Its phase angle is equal to that of the reflector; its magnitude can be controlled by the transmission coefficient \( t \) of the power divider.

One obvious method of making the three necessary wave combinations is a sequential switching of three different reflectors, as shown in Fig. 27. Such a SPR has been constructed and tested, operating satisfactorily in 1 to 700 MHz frequency range.

A different combination circuit has been conceived, avoiding the need of switching (Fig. 28). Essentially, it is a three-fold bridge with tripled left-hand pair of arms (each arm containing one reflector), and three associated detectors. The mutual cross-coupling between the reflectors and non-associated detectors is eliminated by introducing the compensation impedances \( Z_c \), equal to the detector impedances. Such a combination circuit is actually a highly symmetrical structure as shown by its redrawing in Fig. 29. The formula (18) for q-points remains valid.

Being a resistive structure, the SPR has a potential to be very broadband. The ultimate bandwidth limitation is imposed by the ability of the reflectors to maintain three distinct reflection coefficients. It could be three sections of transmission lines with lengths differing nominally by 60°; this would give performance equal to a 3-probe reflectometer (an octave bandwidth). It could be a more sophisticated combination of lines, as proposed by Morawski [24] (a decade bandwidth). However, the broadest possible band is theoretically achieved using lumped-element reflectors (reflection coefficient of an inductor or a capacitor rotates only by 180° over all frequencies). An example for C-L-C reflectors (two capacitors and one inductor) is shown in Fig. 30, displaying the phase angles of their reflection coefficients. Changing an element value results only in shifting the curve along the (logarithmic) frequency axis. The curves should be so positioned as to maximize the minimum of their mutual vertical distances over the required frequency range. It turns out that these minimum phase differences (\( m \) in Fig. 30) are not less than 77° over a 100:1 bandwidth, or 55° over a 1000:1 bandwidth. Multi-element
reflectors (e.g. series or parallel resonance circuits) are even better [29]; moreover, they can accommodate some circuit parasitics.

Fig. 28: Triple-bridge combination circuit

Fig. 29: Triple-bridge combination circuit redrawn
The following is the list of lumped SPR developed and tested:

<table>
<thead>
<tr>
<th>Year</th>
<th>Technology</th>
<th>Nickname</th>
<th>Fmin</th>
<th>Fmax</th>
<th>Application</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>Conventional components</td>
<td></td>
<td>100 kHz</td>
<td>100 MHz</td>
<td>Verification of theory</td>
<td>[30]</td>
</tr>
<tr>
<td>1987</td>
<td>Switched reflectors, conventional components</td>
<td>Gaspar</td>
<td>1 MHz</td>
<td>700 MHz</td>
<td>Verification, general purpose</td>
<td>[31]</td>
</tr>
<tr>
<td>1989</td>
<td>Monolithic GaAs IC</td>
<td>Gaspar</td>
<td>75 MHz</td>
<td>8 GHz</td>
<td>General purpose</td>
<td>[27], [28]</td>
</tr>
<tr>
<td>1990</td>
<td>Thick-film hybrid IC</td>
<td>Hisp</td>
<td>1 MHz</td>
<td>700 MHz</td>
<td>Water level meter</td>
<td>[32]</td>
</tr>
<tr>
<td>1991</td>
<td>Thin-film hybrid IC</td>
<td>Thinx</td>
<td>5 MHz</td>
<td>2 GHz</td>
<td>General purpose</td>
<td>[33]</td>
</tr>
<tr>
<td>1995</td>
<td>Surface-mount</td>
<td>SMS</td>
<td>5 MHz</td>
<td>2.5 GHz</td>
<td>GSM, UMTS antenna installation, fault location</td>
<td></td>
</tr>
</tbody>
</table>

Two of the devices are illustrated in Fig. 31 and Fig. 32.
10. Six-Port Reflectometer versus Conventional Network Analyzer

This section compares selected SPR characteristics against those of conventional network analyzers and lists main advantages and drawbacks of the SPR technique.

**Measurement Accuracy**

Measurement accuracy of a SPR is best expressed in terms of *uncertainty radius* $\delta$ (Fig. 33). It is the radius of a circle centered at measured reflection coefficient inside which the true reflection coefficient lies with a high probability (e.g. 99%). It may combine both systematic and random errors.

A typical value for SPR is 0.01 to 0.03, which, for low reflection coefficient measurement, corresponds to effective directivity 40 to 30 dB. This is comparable with conventional NA. However, SPR used in metrology achieve $\delta$ from $10^{-3}$ to $10^{-4}$.
Phase measurement uncertainty derives from $\delta$ as shown in the figure.

**Measurement Convenience**

Measurement convenience, i.e. the amount of operator’s work to arrive at a result in a desired form with a calibrated instrument, is comparable for SPR and conventional NA.

**Calibration Convenience**

SPR calibration is more involving, requiring more loads to be connected (this is not true for TRL calibration). The disadvantage is compensated for by the fact that calibration is less frequently needed.

Single SPR calibration could in principle be also simplified using computer-controlled electronic calibrators like those used in conventional NA.

**SPR Advantages**

- Simplicity of microwave hardware. This has the following implications:
  1. Long-term stability of operation.
  2. No need for frequent calibration. This is particularly useful in devices integrated in systems.
- Potential to be built as sensors tailored to particular applications.
- Main sources of measurement inaccuracy (nonlinearity, temperature dependence) are concentrated in few localized components (detectors); they can be evaluated and reduced by software.
- No need for phase-locked signal sources. This is useful especially in industrial applications with free-running magnetrons. Yet, in other applications, a synthesized generator is of great advantage for high-accuracy measurements.
- “Unlimited” frequency range. SPR technique can be employed at any frequency for which power sensors exist. This in principle enables to extend frequency range of vector network analyzers up to optical frequencies.

**SPR Drawbacks**

- The main SPR drawback is its broadband nature of signal detection. This has serious implications, common with scalar NA:
  1. Limited dynamic range due to noise. Attenuation measurement with DSPNA is limited to 50–60 dB for the most precision metrological devices as compared to 100–140 dB of heterodyne NA. Measurement process is very slow to achieve the high-end dynamic range of DSPNA. Modulation techniques have been employed to increase the dynamic range.
  2. Sensitivity to spurious signals (e.g. harmonics).
  3. Sensitivity to external interference (e.g. when measuring antenna installations, the antenna under test may receive strong signals from neighboring transmitters, even out of the measurement frequency band).

**11. Applications**

This section lists several applications where six-port reflectometers have been used.

**Metrological Applications**

Metrological applications are what the SPR has originally been developed for. They include

- Metrology of scattering parameters, in particular attenuation
- Metrology of microwave power

The metrological applications benefit from the high stability of SPR due to their simpler construction as compared to other systems. NIST has currently the following devices in use$^2$:

**Coaxial 6-ports**

10–1000 MHz
1–18 GHz

$^2$ Information by the courtesy of J. Juroshek, NIST.
Waveguide 6-ports
18–26.5 GHz (WR-42)\(^3\)
26.5–40 GHz (WR-28)
40–50 GHz (WR-22)
50–75 GHz (WR-15)
75–110 GHz (WR-10)

All of their calibration six-ports are built with thermistor detectors. They are also all dual six-ports.

NIST offers power and scattering parameter calibration services in miscellaneous connector and waveguide sizes. Coaxial SPR are used also for waveguide calibrations, using adapters and calibrating at their waveguide side. Similarly, waveguide SPR are used for coaxial calibrations. The six-ports are calibrated for scattering parameter measurements using the LRL calibration technique and for power by putting a NIST power standard (e.g. thermistor head) on the test port.

However, in recent years they have been transferring some of the S-parameter activities to Agilent HP8510C heterodyne network analyzer. The reason is the cost of calibrating wide-band devices such as 2.4 mm on multiple systems. It is significantly quicker and cheaper to measure those types of devices on the HP8510C.

General Laboratory Applications

Marconi Instruments 6210 Reflection Analyzer offers six-port-based reflection coefficient measurement from 250 MHz to 26.5 GHz. The SPR is built using stripline directional couplers and uses temperature-stabilized diode detectors.

Antenna Installation Tester

A portable battery-operated system exists which combines a synthesized signal generator, a six-port vector reflectometer, a scalar network analyzer, and a spectrum analyzer operating in 10 MHz to 2.5 GHz range (Fig. 34).

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\(^3\) In parentheses US waveguide designation
The system is usable as a general instrument but is mainly applied to mobile communications antenna installation measurements, including time-domain reflectometry (cable fault location).

**High-Power Industrial Applications**

SPR-based instruments for 2450 MHz (up to 30 kW) and 900 MHz (up to 100 kW) exist, which can be supplemented by or integrated with automatic impedance matching systems (an example is shown in Fig. 35). Applications are, among others:

- Microwave heating and drying
- Plasma generation under varying conditions (type of gas, pressure) for the purpose of semiconductor fabrication (e.g. selective etching, layer deposition)
- Thin-film coating of materials (reflectors, sun glasses, optical fibers, metallized plastic sheets)
- Diamond production from CO₂-plasma

![Fig. 35: SPR-based automatic impedance matching system (2450 MHz, 30 kW pulsed or CW)](image)

**Materials Measurement**

SPR is a system of choice when a property of interest can be converted to impedance (reflection coefficient) and using a commercial instrument is too costly. Some reported examples: measurement of complex permittivity, tobacco humidity, contents of unburned coal in ashes.

**Water Level Measurement**

The sensor of the realized water level meter is a transmission line partially immersed in water (Fig. 36). If the line is properly terminated the reflection takes place only at the air-water interface and the phase of reflection coefficient is proportional to the water level. The system used thick-film hybrid-integrated SPR operating at 300 MHz.
12. Perspectives

At the present, the theory of SPR is fully understood and complete, and, as for accuracy, calibration methods are probably in their ultimate state of perfectness. Work continues on developing convenient broadband and noise-proof self-calibration methods. As laboratory instruments, SPR and especially SPNA cannot compete with conventional heterodyne network analyzers up to 100 GHz. Current activities concentrate mainly on developing new SPR structures, an attractive possibility being on-chip measurements by integrated measurement instruments. Application may be e.g. controlling radiation diagrams of multielement antennas or collision-avoidance radars.

Another potential area of application is very high (infrared or even optical) frequencies where the interference-based SPR is readily applicable. However, this has been told over the last three decades.

A group of researchers at Ecole Nationale Superieure des Telecommunications in Paris are still developing and expanding the ideas of lumped SPR [34] – [36]. They presented a hybrid-integrated SPR operating in the band 1.5 MHz to 2200 MHz (bandwidth 1500:1) and a SPR realized in monolithic microwave integrated circuit (MMIC) technology, occupying, including detectors, the surface of 2.2 mm² and operating between 1.3 GHz and 3.0 GHz.

Since 1977, the total number of relevant publications on SPR and related problems has been about 200 (on average 8 papers/year). The rate is at least 4 papers/year over the last 7 years, which means that SPR is still attracting an interest.

13. Conclusions

SPR is now a ripe technology that found itself a definite place among other measurement methods. There are applications where the use of SPR is preferable to other techniques. Conventional network analysis technique has much benefited from SPR calibration theories.
References


