Coaxial 2.45-GHz High Power Impedance Matching Device

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Abstract—**Principle and design of a 2.45 GHz coaxial (1 5/8'' EIA) three-stub impedance transformer (tuner) are presented. Each of the three tuner cells consists of the main coaxial line crossing a waveguide section terminated at one end by a fixed short and at the other end by a sliding short. The paper concentrates on the development of a sufficiently accurate and compact mathematical model of the tuning cell, which is a prerequisite for using the tuner as automatic impedance matching device.**

Keywords impedance matching; tuners; industrial electronics

I. Introduction

Impedance matching in high power microwave installations equipped with coaxial-input applicators would logically prefer *coaxial* tuners as well rather than using the established three stub waveguide tuners [1] with added coaxial-to-waveguide adapters. Coaxial tuner implementations common in low power-techniques are not suitable because they often use open transmission lines or employ, due to insufficient space for microwave chokes, contacting sliding elements, both aspects prohibitive at high powers. A study of an alternative structure that uses a sliding *waveguide* short as tuning element has been reported in [2] with positive results, including experiments with a manufactured prototype. The tuner is composed of three identical *tuning cells* in cascade, separated by odd multiples of a quarter-wavelength. Each tuning cell consists of the main coaxial transmission line traversing a rectangular waveguide section (Fig. 1). The waveguide is terminated at one end by a fixed short with a cutout and at the other end by a sliding short. When moving the sliding short (changing d), the locus of input reflection coefficient $S_{11}(d)$ maps a circle that at certain $d = d_m$ necessarily passes through zero (the cell is then perfectly matched, i.e. completely transparent) and at another $d = d_r$ touches the unit circle (the cell is totally reflecting), which are prerequisites for using such a structure as tuning element.

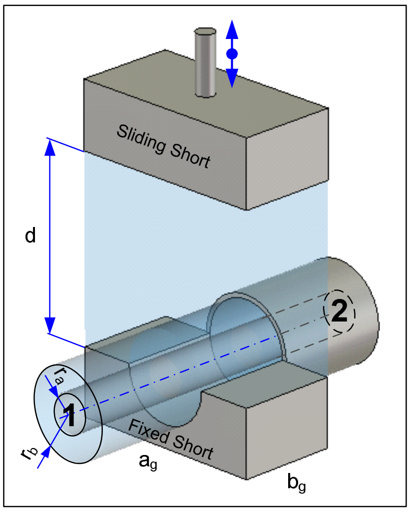

Figure 1. Tuning cell configuration

Based on [3], the cell behavior can be approximated by the equivalent circuit shown in Fig. 2. The waveguide is represented by transmission lines TL1 and TL2, defined by characteristic impedance Z_{0g}, guide wavelength λ_g and lengths d (variable) and d_s (fixed). Both distances are referred to the plane of the coaxial line axis and need not necessarily coincide with physical distances. The coaxial lines are represented by transmission lines TL3 and TL4, defined by their physical characteristic impedance Z_0 (to which also S_{11} and S_{21} are related) and lengths $d_1 = d_2$, measured from the waveguide wall inner planes, represented by terminals 3–3' and 4–4'. The lengths ensure proper electric length of the cell (odd multiples of 90° at the highest frequency of the desired band). Paper [3] provides analytical formulas for the circuit parameters L_1, C_1, C_2, Z_{0g}; all are frequency-dependent. The practical design has been made with 1 5/8'' EIA line (r_a = 8.45 mm, r_b = 19.4 mm) and WR 340 waveguide (a_g = 86.36 mm, b_g = 43.18 mm).

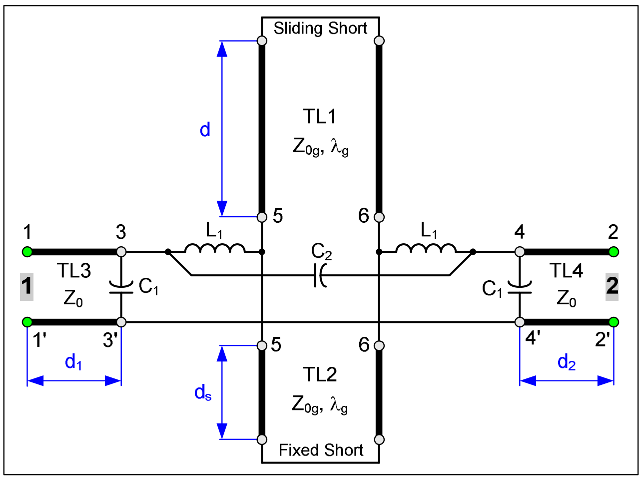

Figure 2. Tuning cell equivalent circuit diagram

Employing a tuning cell in practical automatic impedance matching devices (autotuners) supposes availability of a simple mathematical model which reproduces accurately the cell scattering parameters S_{11}, S_{21} in dependence on sliding short position d and frequency f. The conceptually easiest and straightforward method is direct approximation of the S-parameters by (2-dimensional) polynomials [4]. The weakness of the approach stems from the fact that due to resonances there are intervals of f and d where the slope of the functions changes very rapidly; consequently, high degrees of approximating polynomials are needed. This has several disadvantages, the most serious being the fact that such polynomials, while tending to fit at the input data points, oscillate, leading to unacceptable deviations between the points. Although a solution on the verge of applicability has been arrived at (with help of a specific pre-transformation of the input S-parameters), this method proved unsound for real life use.

Another alternative is using the full equivalent circuit according to Fig. 2. The main advantage compared to direct S-parameters approximation is that even constant or slowly varying circuit parameters naturally generate the rapid resonant variations in S_{11} and S_{21}. This overcomes the main difficulty associated with the previous method. However, there are drawbacks. The analytical formulas for the circuit parameters are only approximation not accurate enough for practical use. Thus, their fitting to actual S-parameters is inevitable. This introduces dependence not only on frequency but also on short position d. Also, due to the circuit complexity, the fitting is not a trivial task and cannot be performed analytically: some sort of optimization is needed. Then, logically, a question arises whether a simpler, better tractable circuit with less variables would not serve equally well.

The present paper proposes such a simplified equivalent circuit, develops a method of arriving at its circuit parameters, and implements their polynomial approximation. The main advantages are that merely two of the circuit parameters are enough for fitting and that the fitting process is analytical and explicit. The moderate costs of the simplification are increased variation of the circuit parameters with frequency and the introduced dependence on the sliding short position. Still, the functions are well approximable by polynomials. The result is a compact tuner cell model consisting of typically 35 numbers (polynomial coefficients). The following sections are devoted to the explaining of the method and its verification by an example case where S-parameters obtained by electromagnetic simulation of the structure have been used as the input data.

II. Simplified Equivalent Circuit Model

The proposed equivalent circuit is shown in Fig. 3. The sliding short is represented by cascaded transmission lines TL1, TL2, both characterized by the guide wavelength λ_g of the used waveguide and the characteristic impedance [3]

$$Z_{0g} = \frac{2 b_g \eta}{a_g \sqrt{1 - (f_c / f)^2}}$$

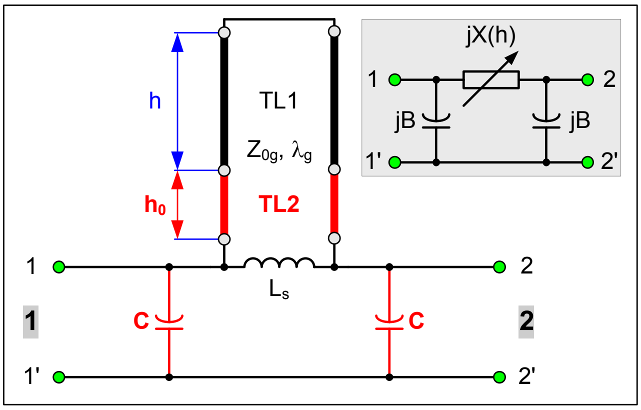

Figure 3. Simplified equivalent circuit of the tuning cell

where a_g and b_g are the waveguide broadside and narrowside widths, $\eta = \mu_0 c \approx 377\ \Omega$ is free space impedance, $\mu_0 = 4\pi \times 10^{-7}$ H/m is vacuum permeability, $c \approx 3\times10^8$ m/s is speed of light, $f_c = c/(2a_g)$ is the waveguide cutoff frequency, and f is frequency. Length $h = d - d_m$ of TL1 is equal to the difference of physical shorting plunger distance d (Fig. 1) and distance d_m at which the match ($S_{11} = 0$) is attained. The fixed short is substituted by inductance L_s (in our case $L_s = 2.82$ nH). The circuit refers to the waveguide wall inner planes.

Only two parameters are used for fitting: capacitance C and length h_0 of the auxiliary *correction* line TL2 (both accentuated red in Fig. 3). The task is to find such polynomial functions $C(h,f)$ and $h_0(h,f)$ that would approximate S_{11} and S_{21} of an actual tuning cell for the corresponding sliding short positions $d = h + d_m$ and the same frequencies f. It will turn out from the nature of the method that C is only function of f.

The circuit can be classified as reciprocal lossless longitudinally symmetrical two-port. For such circuits, S-parameters are constrained as follows: $S_{12} = S_{21}$, $S_{22} = S_{11}$, $|S_{11}|^2 + |S_{21}|^2 = 1$, $\varphi_{21} = \varphi_{11} - \pi/2 \pm n\pi$, where φ_{21} and φ_{11} are phase angles of S_{21} and S_{11}, respectively, and n is integer (in our circuit $n = 0$ or $n = 1$). Consequently, apart from the uncertainty in n, the S-matrix is completely defined by merely two real numbers, e.g. by components of S_{11}. In reality, due to errors, the constraints in measured values of S_{11} and S_{21} are violated; we can deem the quantities uncoupled and use them independently for obtaining two sets of the circuit parameters that can be appropriately averaged.

Fig. 3 is a π-network with shunt susceptance B and series reactance X (see the inset). It is useful to normalize B and X with respect to coaxial line impedance Z_0 as $b = BZ_0$, $x = X/Z_0$; then

$$b = 2\pi f C Z_0 , \tag{1}$$

$$\frac{1}{x} = \frac{Z_0}{2\pi f L_s} + \frac{Z_0}{Z_{0g}} \tan^{-1}\left[2\pi (h + h_0)/\lambda_g\right]. \tag{2}$$

$$S_{11} = \frac{x - 2b + b^2 x}{x + 2b - b^2 x + 2j(bx - 1)}, \tag{3}$$

$$S_{21} = \frac{-2j}{x + 2b - b^2 x + 2j(bx - 1)}. \tag{4}$$

The first step of the proposed procedure will be determining of b and x from either (3) or (4); the two solution pairs will be distinguished by corresponding subscripts mn = 11 or 21. Subsequently, C and h_0 will be obtained from the inverse of (1) and (2). Let $1/S_{11} = u_{11} + jv_{11}$, $1/S_{21} = u_{21} + jv_{21}$. Then the two pairs of (ideally identical) solutions are given by

$$b_{mn} = \frac{v_{mn} \pm \sqrt{u_{mn}^2 + v_{mn}^2 - 1}}{u_{mn} + 1}, \tag{5}$$

$$x_{11} = \frac{2b_{11}(u_{11} + 1)}{u_{11}(b_{11}^2 + 1) + b_{11}^2 - 1}, \tag{6}$$

$$x_{21} = \frac{1 - u_{21}}{b_{21}}. \tag{7}$$

Using (5) for computing b brings two problems. Firstly, the equation is extremely sensitive to input data error when $|S_{mn}| \approx 1$ (the square root approaching zero), leading to glitches in b and x. Secondly, $u_{mn} = -1$ for all S_{mn} that lie on the circle with radius 0.5 and center $(-0.5 + j0)$, which results in singularity in b. Fortunately, b can be found using a different approach. If X (Fig. 3) is varied while B is kept constant (e.g. by changing h at constant frequency), S_{11} moves along a circle with radius 0.5 (circle **k** in Fig. 4) passing the origin (point M, representing the match) and touching unit circle **u** at point R. Point R represents the case of total reflection, i.e. parallel resonance of X, i.e. $|X| \to \infty$. In this case the normalized input admittance is jb hence $S_{11} = (1 - jb)/(1 + jb)$ and phase angle $\alpha = \arg(S_{11}) = -2\arctan(b)$. As a result, b can be obtained from α as

$$b = -\tan(\alpha/2) \tag{8}$$

One actually does *not* need to locate the resonance R to arrive at α. Instead, by noting that α is also the phase angle of the center C of circle **k**, an alternative procedure is possible. For each measurement frequency f_j, ($j = 1, 2, \ldots n_F$) repeat the following steps: (i) Fit a circle to points S_{11} obtained for various short positions h_i ($i = 1, 2, \ldots n_H$) at that frequency. (ii) Determine phase angle α of the center of this circle. (iii) Use (8) to obtain b and, for each h_i, use (6) to obtain x. (iv) Apply the inverse of (1) and (2) to arrive at C and h_0.

A solution supplementary to that obtained from S_{11} can be acquired from S_{21} based on the following consideration. The constraints interrelating S_{11} and S_{21} imply that S_{21} moves along the same circle **k** as S_{11}, being always phase-shifted by either +90° or -90° (points M, S_{11}, S_{21} in Fig. 4 form a right triangle). When S_{11} crosses M, S_{21} crosses R and vice versa. Thus the procedure outlined above can be also applied on S_{21} with the only modification that x is obtained from (7) rather than (6). Averaging the two solutions improves the model robustness.

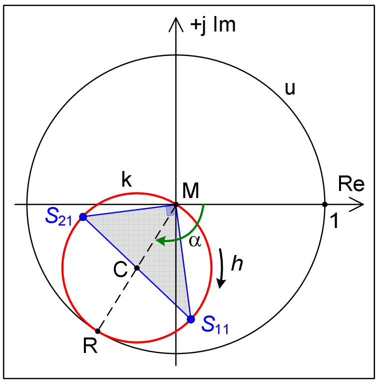

Figure 4. Contours of S-parameters when varying series reactance X

III. Approximation

The set of values

$C[h_i, f_j]$, $h_0[h_i, f_j]$ ($i = 1, 2, \ldots n_H$, $j = 1, 2, \ldots n_F$)

obtained by the explained procedure for each combination of the n_H sliding short positions and n_F frequencies serves as input for the process of polynomial approximation, which results in compacting the model and further reducing the effect of fluctuating inputs. The methodology is similar to that described in [5] for a tuning post in the rectangular waveguide.

As the first step, the variables h and f should be suitably normalized. The normalized sliding short position has been conveniently defined as $H = h/a_g$ and the normalized frequency as $F = 100(f - f_n)/f_n$ where f_n is a chosen *nominal* frequency (in our case 2.45 GHz). Next, the input data are sorted so as to represent a family of n_F tabularly defined functions of H

$Y_j = Y_j[H_i]$ ($i = 1, 2, \ldots n_H$, $j = 1, 2, \ldots n_F$)

where Y stands for C or h_0. The functions are least-square approximated by polynomials of degree D_H (C is actually constant so for it $D_H = 0$)

$$Y_j(H) = \sum_{m=0}^{D_H} a_{mj} H^m \quad (j = 1, 2, \ldots n_F)$$

(H-polynomials). The result is a set of $D_H + 1$ polynomial coefficients a_{mj} for each frequency F_j; these can be looked on as tabulated functions of frequency

$a_{mj} = a_m[F_j]$ ($j = 1, 2, \ldots n_F$, $m = 0, 1, 2, \ldots D_H$)

The functions $a_m[F_j]$ are least-square approximated by polynomials of degree D_F

$$a_m(F) = \sum_{n=0}^{D_F} c_{mn} F^n \quad (m = 0, 1, 2, \ldots D_H)$$

(F-polynomials). The matrix of the coefficients

$[c_{mn}]$ ($m = 0, 1, 2, \ldots D_H$, $n = 0, 1, 2, \ldots D_F$)

then fully defines the corresponding parameter $Y(H, F)$ for any frequency f and any short position h from within the definition range. Matrix $[c_{mn}]$ is the final result of the method.

IV. Example

For verification, the structure in Fig. 1 was simulated by CST Microwave Studio. The matched state occurred at $d = d_m = 15.95$ mm. The simulation was carried out for $h = d - d_m = 0, 2, \ldots 90$ mm ($n_H = 46$); S_{11} and S_{21} were extracted for $f = 2425, 2426 \ldots 2475$ MHz ($n_F = 51$). By the procedure outlined in Section II, values of C and h_0 were computed for each combination of h and f, then sorted to form functions of positions h for each frequency and subjected to approximation. Capacitance C does not depend on h and almost linearly increases with f. A satisfactory approximation was found to be

$$C = 2.374 + 0.0561\, F - 0.0014\, F^2 \qquad (9)$$

The correction length h_0 is a quasi-periodic function of h with period equal to $\lambda_g/2$. Such functions are difficult to approximate by polynomials. Fortunately, to cover all S_{11} magnitudes, values of h between 0 and 72 mm are sufficient. In this limited range the curves can be nearly perfectly approximated by 5-th degree polynomials as shown in Fig. 5. Frequency dependence of the coefficients were approximated by polynomials of typically 4th degree. For illustration, h_0 is fully determined by the following array of coefficients:

c_{mn} for h_0	n = 0	n = 1	n = 2	n = 3	n = 4
m = 0	3.989	-11.396	0.291	0.438	0.000
m = 1	98.095	19.557	-22.583	-4.921	1.575
m = 2	-693.916	14.741	133.456	16.674	-11.449
m = 3	1528.494	-114.290	-329.017	-27.225	30.926
m = 4	-1710.717	150.650	372.731	23.686	-35.490
m = 5	772.186	-52.320	-157.314	-8.949	14.706

Using (9) and the table data, C and h_0 were computed and, in turn, the S-parameters by (1) – (4). Approximation error has been evaluated as distance in complex plane between the original and model S-parameter values corresponding to the same h and f. The result for S_{11} is shown in Fig. 6 (the case for S_{21} is nearly identical). For the most part, the error is below 0.02, which is a value comparable or lower than typical measurement uncertainty. For values of h between 0 and 6 mm the error increases to still acceptable 0.06; the cause is yet to be investigated. The inset illustrates S_{11} and S_{21} for h = 70 mm. The corresponding S_{11} approximation error is the red curve in the main diagram.

Conclusions

Principle and construction of the waveguide sliding-short-based coaxial three-stub tuner have been presented with reference to previous work. Alternatives have been discussed for creating a mathematical model of its tuning cells. A simplified equivalent circuit model of the tuning cell has been proposed; a robust analytical method for obtaining its circuit parameters has been developed. A method of polynomial approximation of the circuit parameters has been designed. In this way, a compact, robust, and accurate tuning cell model has been created. The model has been verified with input data obtained by electromagnetic simulation of the structure.

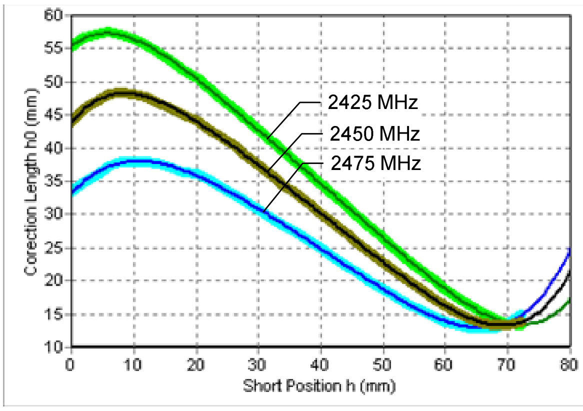

Figure 5. Functions $h_0(h)$ for selected frequencies (thick light lines) and their approximation by 5th-degree polynomials (thin lines)

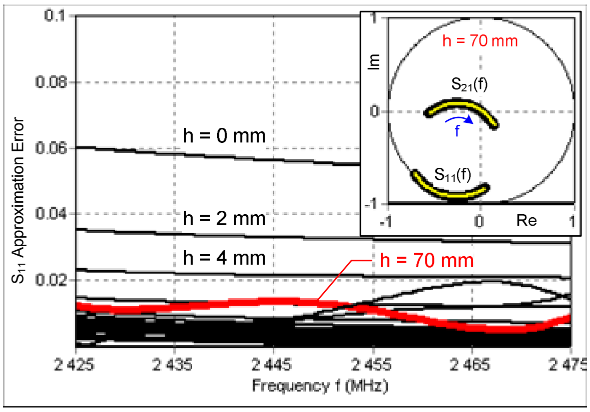

Figure 6. $S_{11}(f)$ approximation error for all h. Inset: input S-parameters (thick black arcs) and model (light inner lines) at h = 70 mm

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